Optimal Green Certificate Auction Design Embedding Economic Dispatch

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ABSTRACT

The rapid development of carbon capture technology speeds up its industrialization and wide application with the help of massive investment. In addition to the capital market, such investment may also come from a well-designed carbon market. This paper proposes a green certificate auction to maximize the auction revenue for enabling the carbon capture technology. Besides political and regulatory requirements, the goodwill from contributing to carbon neutrality may also incentivize the generating companies to participate. The auction design is challenging as it associates with the economic dispatch procedure in the electricity market. Using the notion of virtual demand, we decouple the auction from economic dispatch, and we prove that our designed auction enjoys optimality, truthfulness, and individual rationality. We also show that our auction can be extended to the multi-period scenario, highlighting the impact of leftover certificates. We further provide an upper bound for sample complexity when the willingness of participants cannot be well-identified. Numerical studies verify the effectiveness of the proposed auction and the tightness of the derived sample complexity bound.

CCS CONCEPTS

• **Applied computing** → *Multi-criterion optimization and decision-making*.

KEYWORDS

Green Certificate, Auction Design, Economic Dispatch, Sample Complexity

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1 INTRODUCTION

Global warming is here [1]. The Paris Agreement [2] calls for greater worldwide efforts to combat global warming by reducing carbon emissions. It is recently reckoned by the United Nations Intergovernmental Panel on Climate Change (PCC) that the global carbon emissions must be reduced by 50% by year 2030 to limit warning to 1.5 °C and avoid catastrophic climate consequences [3]. To achieve this goal, carbon tax and cap-and-trade programs are ready in many countries for the large-scale implementation. However, such policies often make the regions with them in a non-favorable position in the global economy. More importantly, the public often challenges the actual usage of the extra payment collected from these carbon-related policies [4]. Thanks to the successful commercialization of carbon capture technologies [5], one potential solution to align the interests between the public and the policymakers is to invest the extra payment in carbon capture technologies. Regarding this analysis, our work designs the green certificate auction to maximize the revenue for the auctioneer (i.e., the system operator in the power grid).

Specifically, we consider the green certificate auction design coupled with economic dispatch (ED), a classic procedure in the electricity sector to dispatch the generators to meet the real-time demand. The designed auction determines the social allocations of green certificates, which grants the generating companies certain advantages in the ED process. Such coupling complicates the auction design. In this paper, we propose the notion of virtual demand to decouple the auction from ED.

1.1 Related Works

We identify two closely related research streams. The first one investigates the applications of financial instruments to the carbon market, while the other one is the theoretical treatment for homogeneous divisible-good's auctions.

Various financial instruments have been implemented in the carbon-related markets, e.g., auction, grandfathering, uniform or discriminatory pricing [6]. We focus on the auction design, the most popular form among its rivals [7]. Both sealed and dynamic auctions have been adopted to allocate green certificates [8]. For example, Betz *et al.* [9] propose an ascending clock auction to improve the efficiency of the green certificate market. Wang *et al.* [10] employ the sequential ascending auction and prove its convergence to the Pareto optimal equilibrium. Rao *et al.* [11] study the uniform price sealed auction and show the existence of an asymmetric Nash Equilibrium. Sun *et al.* [6] generalize the setting by considering the multi-buyer and multi-seller scenario and design a double action for green certificate allocation. Ding *et al.* [12] consider the influence of the interactions between the energy consumption market and the certificate auction market and design a two-stage auction-bargaining model. Wang *et al.* [13] design the multi-unit auction in the Bayesian framework. Overall, the literature rarely treats the problem as a divisible-good auction and seldom involves rigorous modeling of the whole dispatch process. Furthermore, the literature often designs the market for the policymaker instead of the system operator in the electricity sector. We seek to bridge the gap by rigorously designing the green certificate auction embedding the ED process for the system operator.

We cast the green certificate auction in the homogeneous divisible-good auction design framework since the green certificates are homogeneous. Such an auction is often organized in two ways. The first approach is to discretize the quantity space to a countable number [14]. However, the effective auction design is a long-standing open problem when the number of pieces resulting from the division is too large. The second approach pioneered by Wilson [15] designs effective bidding in a stylized model. This classical work applies to the auction of divisible goods [16, 17], ignorance of the difference between uniform and discriminatory auction. Back *et al.* [18] further show that discriminatory auction yields more revenue in divisible-good auction. Recently, Lu *et al.* [19] design the divisible unit good auction with budget constraints. Johari *et al.* [20] propose a scalar strategy for Vickrey-Clarke-Groves (SSVCG) mechanism and introduce an efficient algorithm to characterize the Nash Equilibrium. However, most of these works assume the knowledge of the bidders' value distributions. Sample complexity [21] is proposed to relax this strong assumption. Both Dhangwatnota *et al.* [22] and Cole *et al.* [23] study the sample complexity for digital goods with an unlimited supply. In this work, we first follow the classical framework proposed by Maskin *et al.* [16] and then use the notion of sample complexity to infer the generating companies' valuation with a limited supply of green certificates.

1.2 Our Contributions

Based on the literature review, we mainly solve two challenges in this paper. First, it is rather challenging to combine an auction with another optimization problem, as we need to consider a multi-object problem for the auction, where the auction outcomes will influence the other optimization. Furthermore, we need to guarantee incentive compatibility, individual rationality, and optimality in this special auction problem. We expensively extend the tradition multi-demand auction framework in [24] to our problem, by analyzing the monotonicity characteristics in our problem, which are never studied in this setting. The second challenge is how to identify the sample complexity in this multi-demand auction problem. Conventionally, the sample complexity is discussed in the single item auction [25] and non-linear pricing with infinite supply [23]. We are the first to study sample complexity in this problem. This is no easy task. It is challenging to characterize the bound for sample complexity due to the coupling in this multi-demand auction. We propose an auxiliary mechanism to derive an effective bound.

Overall, our principal contributions are as follows:

(1) *Virtual Demand*: We study the interdependency between the green certificate auction and the ED process. Inspired by the notion of virtual value [26], we propose a new concept-the virtual demand-to decouple the interdependency.
(2) *Theoretical Insights*: Our designed auction in the Bayesian framework is both truthful and individually rational. We further investigate the impact of incorporating capacity constraints (to be explained in detail in Section 4) into our auction design, yielding the insights for both competitive and non-competitive scenarios.
(3) *Sample Complexity*: We construct an upper bound for the number of samples needed to estimate the value of bidders accurately. This bound informs us when the auction design can achieve guaranteed performance.

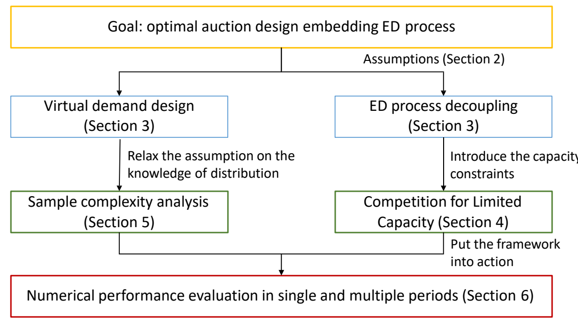


Figure 1: The structural paradigm of our paper

The rest of the paper is organized as follows. We first formulate the green certificate auction design problem in Section 2, which is the basis for our designed auction in a constraint-free setting. We theoretically prove the effectiveness of our design in Section 3 and then extend it with capacity constraints in Section 4. We conduct a sample complexity analysis to relieve the assumption on the knowledge of bidder value distribution in Section 5. After that, numerical studies in Section 6 are conducted to verify our conclusions, followed by concluding remarks in Section 7. Fig. 1 illustrates the structural paradigm of our paper.

2 AUCTION FORMULATION

We consider the optimal multi-unit auction for the green certificates, assuming the green certificates are divisible, with a total amount of Q. The bidders in the auction are the generating companies, whereas the auctioneer is the system operator. Denote the total number of bidders by N and the type of generator i by v_i. This value information characterizes the company's willingness to hold the certificates. The potential benefit may come from future trading, goodwill, or other side rewards beyond this mechanism. Specifically, denote the auction outcome for generator i by variable x_i. We focus on studying the uniform demand price function, i.e., the price $r(x_i, v_i)$ is fully characterized by the auction outcome x_i and type v_i. Note that the integral of $r(x_i, v_i)$ with respect to x_i describes the valuation for generator i. Mathematically, if the realized auction outcome for generator i (green certificate purchased by the generator through the auction) is q_i, then its valuation $\Psi(q_i, v_i)$ is determined as follows:

$$\Psi(q_i, v_i) = \int_0^{q_i} r(x, v_i)dx. \quad (1)$$

We further assume the type information v_i is a random variable, drawn from the Cumulative Probability Function (cdf) $F_i(v)$ with a support of $[\underline{v_i}, \overline{v_i}]$. Denote its corresponding Probability Density Function (pdf) by f_i.

To simplify the optimal auction design, we make the following technical assumptions:

- **A1**: All the value distributions of bidders are regular. That is, for each generator i, the virtual valuation function $J_i(v) = v - \rho_i(v)^{-1}$ is increasing. Note that $\rho_i(v)$ represents the hazard rate for the bidder i:

$$\rho_i(v) = \frac{f_i(v)}{1 - F_i(v)}. \quad (2)$$

- **A2**: For each type v, $r(x, v)$ is finite and positive, twice continuously differentiable, strictly decreasing in x, and strictly increasing in v.
- **A3**: The elasticity of the demand price function is non-decreasing, i.e.,

$$\frac{\partial}{\partial v}\left(-\frac{x}{r}\frac{\partial r}{\partial x}\right) \leq 0. \quad (3)$$

- **A4**: The demand price function is concave in v:

$$\frac{\partial^2 r}{\partial v^2} \leq 0. \quad (4)$$

In classical Myerson's auction, Assumption A1 often guarantees the monotonicity of the auction, and incentive compatibility of the auction [26]. The remaining three assumptions are standard technical assumptions for demand price functions. A wide range of functions satisfies the four assumptions [24].

These four assumptions are essential in characterizing the objective functions for the bidders and the auctioneer in the green certificate auction.

Assume the marginal cost of generator i to be α_i. Denote the total demand by d and the generation of generator i by g_i, which is bounded by the green generation capacity. Specifically, we denote the green generation capacity by B_i without purchasing green certificates in an emission-aware ED. The green certificate grants the generators more opportunities to get dispatched in ED. For example, if generator i, through the auction, obtains q_i amount of green certificates, its maximal generation level becomes $B_i + q_i$.

Remark: We want to emphasize that in the ED process with green certificates, each unit of electricity can only be allowed to support the demand through ED if the associated generator could present the corresponding green certificate for this unit of electricity. This is the key difference between the classical ED process and the ED process with green certificates.

Thus, the system operator, based on the outcome of the auction, could conduct the ED by solving the following optimization problem:

$$\begin{aligned} \text{(P1)} \ \min \ & \sum_{i=1}^{N} \alpha_i g_i \\ s.t. \ & \sum_{i=1}^{N} g_i = d \\ & 0 \leq g_i \leq B_i + q_i \ \ \forall i. \end{aligned} \quad (5)$$

The first constraint ensures the supply-demand balance. The second set of constraints refers to the generation emission constraints. Problem (P1) decides the energy price λ, the Lagrangian multiplier associated with the supply-demand balance constraint. Define λ_0 to be the energy price without the green certificate auction (i.e., all the q_is are zero).

Next, to ensure the existence of a feasible solution, we make Assumption A5 as:

- **A5**: When all the certificates are released, the demand d must be satisfied with all the generators, i.e.,

$$d \leq \sum_{i=1}^{N} B_i + Q. \quad (6)$$

Thus, we can characterize the change in dispatched generation (denoted by Δg_i) due to the introduction of auction as a function of the optimal auction outcome q_i^* and the energy prices λ_0 and λ. Specifically,

$$\Delta g_i = \begin{cases} q_i^* & \alpha_i < \lambda, \\ -B_i & \lambda \leq \alpha_i < \lambda_0, \\ 0 & \alpha_i \geq \lambda_0. \end{cases} \quad (7)$$

Eq. (7) allows us to derive the ED profit change for the generator. Specifically, we can express the extra profit for generator i by participating in the auction as follows:

$$\begin{aligned} V_i &= \Delta g_i(\lambda - \alpha_i) + (g_i - \Delta g_i)(\lambda - \lambda_0) \\ &= \mathbf{I}(\alpha_i < \lambda)\lambda(g_i - B_i) - \mathbf{I}(a_i < \lambda)\alpha_i(g_i - B_i) \\ &\quad + \mathbf{I}(\alpha_i < \lambda)(\lambda - \lambda_0)B_i - \mathbf{I}(\lambda \leq \alpha_i < \lambda_0)\lambda_0 B_i \\ &\quad + \mathbf{I}(\lambda \leq \alpha_i < \lambda_0)\alpha_i B_i \\ &= \mathbf{I}(\alpha_i < \lambda)(\lambda - \alpha_i)(g_i - B_i) + \phi(\lambda, \lambda_0, \alpha_i), \end{aligned} \quad (8)$$

where $\mathbf{I}(\cdot)$ is the indicator function. The extra profit comes from two components: the first one is the profit from the additional generation (i.e., Δg_i) and the second one is the profit difference of the generation due to price change (i.e., $g_i - \Delta g_i$). Characterizing the ED process deepens our understanding of the auction design, as these two processes closely couple together. From the ED, the objective of the system operator is to minimize the extra payment for the certificates in ED while maximizing the auction revenue. We represent this objective function as follows:

$$\max \ \ \mathbf{E}_{v_i}\left[\sum_{i=1}^{N} p_i - \sum_{i=1}^{N} V_i\right], \quad (9)$$

where p_i denotes the payment that bidder i pays for q_i green certificates.

Each generator's utility function, denoted by U_i, consists of two components: one is the utility extracted from the auction, and the profit from participating ED[1]:

$$U_i = \Psi(q_i, v_i) - p_i + V_i. \quad (10)$$

We assume that if participating in the auction does not provide the generator with any extra profit ($U_i < 0$), it will opt out the auction.

[1]We assume that for generator i, whose marginal cost equals the price, it will not get dispatched in ED.

3 REVENUE-MAXIMIZING AUCTION DESIGN

The formulated auction is a variant of the Myerson auction. Compared with the classical Myerson auction, our problem associates the ED problem with the auction outcome. To decouple the two processes, we first find a more direct formulation of the ED problem. In our formulation the final solution λ comes from a discrete set $\{\alpha_i, i \in [N]\}$.

Note that for a given λ, the best choice of g_i for generator i is as follows:

$$g_i = (B_i + q_i^*)\mathbf{I}(\lambda \le \alpha_i). \tag{11}$$

Eq. (11) allows us to express the utility function U_i^λ of generator i as a function of ED price λ as follows:

$$U_i^\lambda = \begin{cases} \Psi - p_i + \phi(\lambda, \lambda_0, \alpha_i), & \lambda \le \alpha_i. \\ \Psi - p_i + (\lambda - \alpha_i)q_i + \phi(\lambda, \lambda_0, \alpha_i), & \lambda > \alpha_i. \end{cases} \tag{12}$$

Note that Ψ is a function of q_i and v_i, i.e., $\Psi(q_i, v_i)$. We omit the parameters for neat expression when there is no confusion. Then we propose to define the virtual demand function I^λ for our auction:

$$I^\lambda(q_i, v_i, \alpha_i) = \begin{cases} \Psi - \dfrac{1}{\rho_i(v_i)}\dfrac{\partial \Psi}{\partial v_i}, & \lambda \le \alpha_i. \\ \Psi - \dfrac{1}{\rho_i(v_i)}\dfrac{\partial \Psi}{\partial v_i} - \alpha_i q_i, & \lambda > \alpha_i. \end{cases} \tag{13}$$

Remark: The virtual demand design is inspired by the classical Myerson auction and some facts observed from the auction with ED process: the generators that get dispatched will have low incentive if their generation costs are high; the incentive of generators which have not been dispatched due to high costs will not be influenced by the ED process. That's also the intuition of our auction design.

Next, we introduce Assumption A6 to facilitate the subsequent analysis.

- **A6**: (Rare Item Assumption) For all v_i profiles, define $q_i^{'}$ as the quantity to make $\partial I^\lambda(q_i, v_i, \alpha_i)/\partial q_i = 0$ when $\lambda > \alpha_i$. We further assume that the following condition holds $\sum_{i=1}^N q_i^{'} \ge Q$.

Remark: This assumption ensures that all the bidders will compete for the certificates. More specifically, with Assumption A6, there always exists i that makes $\partial I^\lambda(q_i, v_i, \alpha_i)/\partial q_i$ positive, meaning that the bidder starves for more certificates. We will relax the assumption by adding the capacity constraints in the subsequent analysis, which means more certificates than necessary are clipped, and the competition among generators becomes less fierce.

Now we are ready to design the optimal multi-unit auction. In our framework, the bidder (generation company) bids its own v_i to the system operator, who provides a recipe (p_i, q_i) for the allocation and payment.

We construct the optimization problem to derive the best allocation and payment as the benchmark:

$$\begin{aligned} \text{(P2)} \ \max_{q_i} & \sum_{i=1}^N I^\lambda(q_i, v_i, \alpha_i) \\ s.t. \quad & \sum_{i=1}^N q_i \le Q \end{aligned} \tag{14}$$

The Karush–Kuhn–Tucker (K.K.T.) conditions [27] yield the characterization of the optimal allocation q_i^*s:

$$\begin{aligned} q_i^* \left[\frac{\partial I}{\partial q_i} - \mu\right] = 0, \text{ and } & \sum_{i=1}^N q_i \le Q \\ & q_i^* \ge 0 \\ q_i^* = 0 \to \frac{\partial I}{\partial q_i}(0, v_i) \le \mu, & \end{aligned} \tag{15}$$

where μ is the multiplier for the constraint $\sum_{i=1}^N q_i \le Q$. The multipliers for $q_i^* \ge 0$ can be cancelled out by directly discussing the scenarios when multipliers equal or do not equal 0.

Based on q_i^* characteristics, we can further express the certificate payment p_i as follows:

$$\begin{aligned} p_i = & \Psi - \frac{1}{\rho_i(v_i)}\frac{\partial \Psi}{\partial v_i} \\ & + \mathbf{I}(\alpha_i < \lambda)(\lambda - \alpha_i)q_i^* + \phi(\lambda, \lambda_0, \alpha_i) \end{aligned} \tag{16}$$

More specifically, with q_i^*, we can calculate $\Psi(q_i^*, v_i)$ based on bid v_i. We can also derive $\frac{1}{\rho_i(v_i)}\frac{\partial \Psi}{\partial v_i}$. Then we can use q_i^* to conduct the ED to obtain λ with auction. Finally, we use the expression ϕ in Eq. (8) to get the final payment in the auction.

To achieve the optimal allocation, we design the auction mechanism described in Algorithm 1.

Algorithm 1 ED-Embedded Green Certificate Auction

Input: The generator i's cost α_i $\forall i \in [N]$;
The generator i's bidding type v_i $\forall i \in [N]$;
Output: The final allocation q_i $\forall i \in [N]$;
The price λ for ED process;
The payment p_i $\forall i \in [N]$;
Initialize $R = 0$
for $\lambda \in \{\alpha_1, ..., \alpha_N\}$ **do**
  Solve the optimization problem Eqs. (14) using Eqs (16) and (15) to derive $(q_i^\lambda, p_i^\lambda)$ $\forall i$.
  Use final auction allocation results and price λ to conduct ED process
  if d is exactly satisfied **then**
    Calculate the final revenue R_t for auctioneer;
    $q_i = q_i^\lambda, p_i = p_i^\lambda$ $\forall i$; $R = R_t$
    return (q_i, p_i) $\forall i$; λ;
  end if
end for

The essence of this algorithm is to conduct an optimal multi-unit auction for the certificates. We first assume a fixed λ and figure out the associated revenue. Since λ is drawn from a discrete set, we can enumerate all possible λs. During the enumeration, we also need to investigate the proposed auction's correctness and effectiveness, guaranteed by the following two theorems.

Theorem 1 Under Assumption A1-A6, there exists a λ that satisfies the demand d.

Proof: We provide a constructive proof.

First, we sort the generators according to the marginal cost α_i. Without loss of generality, we denote the generators by $\alpha_1, ..., \alpha_N$

according to the rank. If for $\lambda = \alpha_K$, we could not satisfy the demand, i.e.,

$$d > \sum_{i=1}^{K-1} B_i + \sum_{i=1}^{K-1} q_i \tag{17}$$

then it's evident that

$$\sum_{i=1}^{K-1} q_i \leq Q \tag{18}$$

When λ increases to $\lambda' = \alpha_{K+1}$, I^λs for the first $K-1$ bidders remain the same. For K^{th} bidder, I^λ needs to be subtracted by $\alpha_K q_K$. From Eqs. (15), we could conclude that the new auction price μ' should not be higher than the original μ. Therefore, if we denote the new allocation for bidder i as q_i^*, we know that

$$\sum_{i=1}^{K-1} q_i^* \geq \sum_{i=1}^{K-1} q_i \tag{19}$$

Thus, $\sum_{i=1}^{K} B_i + \sum_{i=1}^{K} q_i$ increases in K. Recall Assumption A5, which guarantees that the largest possible generation is higher than the demand. Together with the fact that $\sum_{i=1}^{K} B_i + Q$ is strictly increasing in λ, we are guaranteed to find a λ that exactly satisfies demand d. ■

Theorem 1 guarantees that the auction is always feasible. Then we show the effectiveness of our mechanism. Define bidder i's strategy as $s_i(v_i)$, vector s as $[s_1, ..., s_N]$ and vector s_{-i} as $[s_1, ...s_{i-1}, s_{i+1}, ..., s_N]$. Thus, we could define the Bayesian equilibrium profile as $s^* = [s_1^*, ..., s_N^*]$ where $s_i^*(\cdot)$ is equilibrium strategy. Assume that the resulting allocation is deterministic and thus $q_i(x, v_{-i}) = q_i(s_i^*(x), s_{-i}^*(v_{-i}))$ and $p_i(x) = \mathbf{E}_{v_{-i}}[p_i(s_i^*(x), s_{-i}^*(v_{-i}))]$, where x denotes the actual bidding for bidder i. We also denote that $q_{-i}(x, v_{-i}) = [q_1(s_i^*(x), s_{-i}^*(v_{-i})), ..., q_{i-1}(s_i^*(x), s_{-i}^*(v_{-i})), q_{i+1}(s_i^*(x), s_{-i}^*(v_{-i})), ..., q_N(s_i^*(x), s_{-i}^*(v_{-i}))]$.

These notations allow us to express the expected surplus for generator i, denoted by $\Pi_i(x, v_i)$, as follows:

$$\begin{aligned}
&\Pi_i(x, v_i) \\
&= \mathbf{E}_{v_{-i}}[\Psi(q_i(x, v_{-i}), v_i)] - p_i(x) \\
&\quad + \mathbf{E}_{v_{-i}}[(\lambda(q_i(x, v_{-i}), q_{-i}(x, v_{-i})) - \alpha_i)^+ q_i(x, v_{-i})] \\
&\quad + \mathbf{E}_{v_{-i}}[\phi(\lambda(q_i(x, v_{-i}), q_{-i}(x, v_{-i})), \lambda_0, \alpha_i)] \\
&= \mathbf{E}_{v_{-i}}[\Psi(q_i(x, v_{-i}), v_i)] - p_i(x) \\
&\quad + \mathbf{E}_{v_{-i}}[(\lambda(x, v_{-i}) - \alpha_i)^+ q_i(x, v_{-i})] \\
&\quad + \mathbf{E}_{v_{-i}}[\phi(\lambda(x, v_{-i}), \lambda_0, \alpha_i)],
\end{aligned} \tag{20}$$

where $(\cdot)^+$ is the operator $\max(\cdot, 0)$. The expected surplus $\Pi_i(x, v_i)$ is composed of three terms: the expected valuation, the payment, and the expected extra profit in Eq. (8) when generator i's bidding x.

To characterize the effectiveness, we introduce both Bayesian Incentive Compatibility (BIC) and Interim Individual Rationality (Interim IR).

Definition 1 (Bayesian Incentive Compatibility (BIC)) For the best strategy $s_i^*(v_i)$, BIC guarantees that

$$\Pi_i(v_i, v_i) = \max_x \Pi_i(x, v_i).$$

Definition 2 (Interim Individual Rationality (Interim IR)) Interim IR requires that $\Pi_i(v_i, v_i) \geq 0$.

Theorem 2 Our designed auction satisfies both BIC and Interim IR.

To prove this theorem, we first construct the equivalent conditions for BIC and Interim IR in the following Lemma.

Lemma 1 (BIC and Interim IR equivalence) Under the Assumptions A4 and A6, if the auction allocation rule $q_i(v_i, v_{-i})$ is non-decreasing in v_i, the equivalence condition for BIC and Interim IR is that the expected surplus could be expressed as follows:

$$\begin{aligned}
\Pi_i(v_i, v_i) =& \Pi_i(\underline{v}_i, \underline{v}_i) \\
&+ \mathbf{E}_{v_{-i}}\left[\int_{\underline{v}_i}^{v_i} \frac{\partial \Psi(q_i(z, v_{-i}), z)}{\partial z} dz\right]
\end{aligned} \tag{21}$$

Lemma 1 allows us to embed these two notions (BIC and Interim IR) into the objective of optimization for maximizing revenue. We can also show the monotonicity based on the characteristics of the ED process to support our Lemma 1, with details in Appendix A.

Remark: Theorem 2 is remarkable, leading to many exciting and insightful observations on our auction design. We find that I^λ, which reflects the virtual demand for the generator, becomes lower when its cost becomes higher if it is dispatched. It makes sense that a smaller cost in generation will raise the incentive. However, for other cases, the observation is not very intuitive. By diving into the details of the proof (See Appendix B), we realize that the generators that do not get dispatched have higher incentives. Specifically, the system operator extracts extra payments for all dispatched generators from the profits associated with the dispatch. This procedure reduces the incentives of these generators. From the perspective of the system operator, this is highly desirable. Such a process rewards the high-cost generators with more willingness to conduct carbon emission reduction.

From the perspective of economic dispatch, after the possible assignment of green certificate, the electricity prices associated with ED will not increase. When the price does not change, the generators whose cost is lower than the price will generate more if they purchase the certificates. When ED price decreases, some of the generators will be squeezed out for their high costs.

While the proof for Theorem 2 is too long, we provide all the details in Appendix B. Nonetheless, we want to highlight the following insightful Proposition, which is a direct result from the proof for Theorem 2.

Proposition 1 The optimal allocation q_i^* is monotonic in type v_i.

This proposition shows the characteristics of the optimal allocation. The increasing number of one type of participants could jointly drive the ED price closer to the marginal generation cost of this type and obtain more dispatched generation. Hence, the type information will determine the competitiveness in certificate assignment and the associated ED process. Higher type yields more certificate assignment and more generation and revenue in the ED process.

4 COMPETITION FOR CERTIFICATES

This section considers the capacity constraints for the green certificates. As mentioned in the previous section, we consider competitive cases with rare certificate supply specified by Assumption A6. In this section, we consider the capacity constraints that limit the generators' auction outcomes below their physical generation capacities, which significantly affects the characteristics of certificate purchase competition among generators in the auction.

Particularly, the capacity constraint means that the system operator prevents impossible generation outcome with the auction. Note that the green generation limit is the sum of the green generation capacity without certificate B_i and auction assignment q_i. If this limit is higher than the maximal physical generation capacity G_i, the system operator judges that the spare part (i.e., $B_i + q_i - G_i$) would never be used by the generator itself and refuses to allocate this spare part. The capacity constraint will limit the pure green certificate arbitrage among the generating companies.

We modify the optimization problem (P2) with the capacity constraints as follows:

$$\begin{aligned} \text{(P3)} \max_{q_i} & \sum\nolimits_{i=1}^{N} I^{\lambda}(q_i, v_i, \alpha_i) \\ s.t. & \sum\nolimits_{i=1}^{N} q_i \leq Q \\ & 0 \leq q_i \leq G_i - B_i, \forall i \end{aligned} \tag{22}$$

The new constraints allow our mechanism applicable to both competitive and non-competitive scenes.

First, assume that the generators need to compete for the green certificates. In other words, even though the capacity constraints will reduce the willingness for the generators to purchase certificates, the competition remains. In this case, we could conduct the allocation according to optimization problem (P3).

To guarantee the truthfulness, the key is to show that our allocation with (P3) is monotone.

Theorem 3: Under Assumption A6, if we assign the certificates following modified Algorithm 1, which replaces Problem (P2) with (P3), then q_i^* is non-decreasing in v_i.

We extend the proof of Theorem 1 and 2 to prove Theorem 3. We consider the cases when the capacity constraints are active and inactive respectively. Clearly it strictly follows our proofs for Theorem 1 and 2 when constraints are inactive. For active capacity constraints, it can be proved based on the following observation: the generator will keep the maximal generation when v_i increases. The detailed proof is provided in Appendix C.

After discussing the competitive scenes, we modify our mechanism to be adaptive for the non-competitive cases, where $\sum_{i=1}^{N} G_i - B_i < Q$. In this case, we use Eq. (16) for payment design to guarantee truthfulness and rationality. In general, for the optimal auction with capacity constraints, we could modify Algorithm 1 to achieve desirable properties. We summarize the discussions above in Algorithm 2.

Algorithm 2 Auction with Capacity Constraints

Input: Inputs of Algorithm 1;
The generator i's physical capacity G_i $\forall i \in [N]$ and green generator capacity without certificate B_i;
Output: Outputs of Algorithm 1;

```
Initialize R = 0
if ∑_{i=1}^N G_i − B_i < Q then
    q_i = G_i − B_i
    Decide λ through ED process and decide p_i according to Eq. (16).
    return (q_i, p_i) ∀i; λ;
else
    Conduct ED-Embedded Green Certificate Auction (i.e., Algorithm 1) with the optimization problem (P3) and Eq. (16), yielding (q_i, p_i) ∀i; λ;
end if
```

- **A7**: Uniform price function $r(q, v)$ satisfies that $|\frac{\partial r}{\partial v}|$ and $|\frac{\partial^2 r}{\partial v^2}|$ are bounded over the interval $[\underline{v}_i, \overline{v}_i]$.
- **A8**: For the distribution function f_i, we assume $f_i > 0$ over the interval $[\underline{v}_i, \overline{v}_i]$ and f_i is differentiable.
- **A9**: Assume $\rho_i(v_i)^{-1}$ is Lipschitz continuous with parameter L_3.

These assumptions pave the way for characterizing function h, the mapping from $\mathbf{v} = \{v_i, \forall i \in [N]\}$ to $[0, 1]$.

Since $\mathbf{v}$ follows the distribution $F = F_1 \times F_2 \times ... \times F_N$, i.e., all the bidder types are independent, it holds that:

$$h(F) = \mathbf{E}_{\mathbf{v} \sim F}[h(\mathbf{v})]. \tag{23}$$

Suppose the mechanism could only be chosen from a hypothesis class, denoted by $\mathcal{H}$. $OPT_{\mathcal{H}}(F)$ shows the optimal expected revenue as in the following equation:

$$OPT_{\mathcal{H}}(F) = \sup_{h \in \mathcal{H}} h(F). \tag{24}$$

Thus, our samples are from the distribution F. To characterize the sample complexity, we consider E_i as the uniform distribution over i^{th} coordinate of the samples, and we define $E = E_1 \times E_2 \times ... \times E_N$. The sample complexity for our hypothesis class $\mathcal{H}$ is defined to be the minimum number of samples $S(\epsilon, \delta)$ such that for any distribution F, we could find a mechanism h with probability $1 - \delta$ satisfying

$$h(E) \leq OPT_{\mathcal{H}}(F) - \epsilon, \tag{25}$$

where ϵ is a small number in $[0, 1]$.

Now, we conclude the upper bound of the sample complexity for our auction as follows:

Theorem 4: In our proposed Green Certificate Auction with N generators, the sample complexity is upper bounded by $O(\frac{N^2}{\epsilon^3} log \frac{1}{\delta})$ when ϵ is small enough.

We provide the proof in Appendix D.

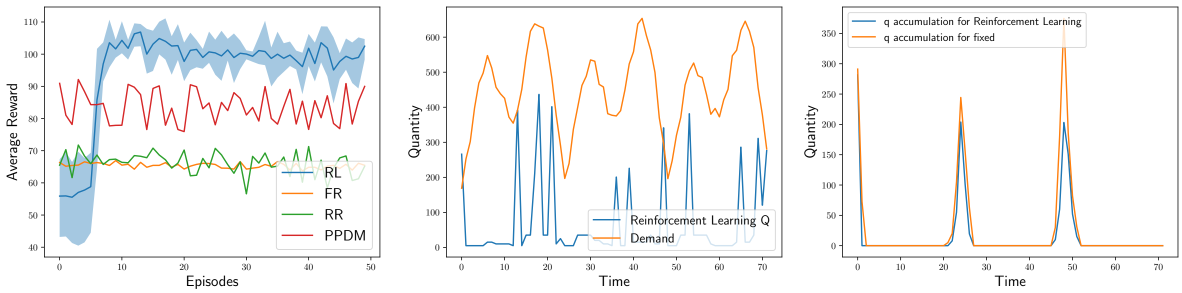
5 SAMPLE COMPLEXITY ANALYSIS

This section focuses on the scenarios when the distribution function for each type v_i is unknown. In this case, we need to learn the value distribution from the historical samples, and we are interested in understanding how many samples are necessary to achieve an approximately optimal auction.

To simplify the analysis, we first reformulate our auction, by expressing the auction as a function h whose inputs are different constant cost α_i and type v_i $\forall i \in [N]$. The maximal revenue is lower than $\max_{q_i} \sum_{i=1}^{N} \Psi(q_i, \overline{v}_i)$, denoted by a constant C_1. We make the following general assumptions on the uniform price function and the distribution functions for v_i.

6 NUMERICAL STUDIES

This section conducts numerical studies to evaluate the performance of our proposed auction design.



(a) The reward for different strategies (b) The strategic certificate release for reinforcement learning (c) The certificate remaining for a single participant

Figure 2: The multi-period auction results

We use the field generator marginal cost data, and capacity data in EIA-860 collected in [28]. The dataset (EIA-860 and EIA-923) that we use is the generation marginal cost and capacity data collected by U.S. Department of Energy, and Energy Information Administration (EIA) for all electric power plants in the United States in 2019. We use the total generation and the annual fuel cost in EIA-923 to estimate the linear marginal cost and we also find the corresponding generator's capacity in EIA-860. We choose 112 generators that are distributed in the same region (Alabama) to conduct our experiments. Then we randomly assign the demand d and the quantity of the total certificates Q, guaranteeing $d \leq \sum_{i=1}^{N} B_i + Q$. The value v_i for each generator i is also set randomly. Specifically, we try to take the values from a certain distribution (uniform or truncated normal). We set function $r(q, v)$ to have the following polynomial form:

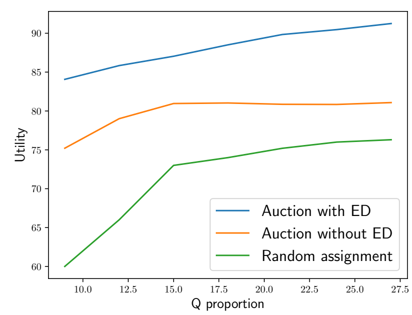
$$r(q, v) = g - a_1 q^2 + b_1 vq - a_2 v^2 - a_3 q + b_2 v. \quad (26)$$

This form satisfies Assumptions A1-A4, and the constant we mentioned in assumptions could also be well calculated.

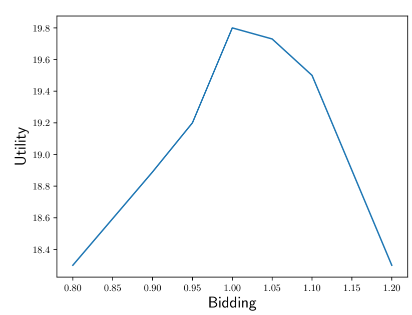
In the following subsections, we conduct two types of numerical studies to examine the performance of our proposed auction. Since our proposed auction focuses on one-shot trading and we have proved its effectiveness, we try to extend it to multi-shot trading. We propose some heuristic strategies to optimize the revenue and verify the performances empirically. Then, we verify the effectiveness of our sample complexity bound for our proposed auction with the empirical distribution.

6.1 Auction verification

We design the experiments for joint ED and auction design. We compare the auction outcomes of three mechanisms: our mechanism, the auction without embedding ED and randomly assignment. The auction without embedding ED means that we first conduct the multi-demand auction in [16], which is similar with the Myerson auction, then we conduct ED process. Randomly assignment means that we randomly assign the certificates, which also satisfies IC. We repeat the experiments for 100 times to derive convincing results. Thus, we can verify the optimality of our proposed mechanism under different total certificate Q levels in Fig. 3(a). We observe the results by changing Q from 9% to 27% of d. We can find that if we do not consider the ED process in the auction, the revenue



(a) The comparison for optimality of our auction design



(b) IC verification of our auction design

Figure 3: Experiments for joint ED and auction design

decreases. We also find the randomly assignment has the worst performance. Then we choose one generator to examine its revenue under different biddings in Fig. 3(b). It can be observed that if it does not bid truthfully, it will suffer a decrease in the revenue.

6.2 Extension to Multi-Shot Scenarios

First, we heuristically generalize our framework to a more realistic scenario with the multi-period auction for each dispatch decision time. The main difference between one-shot auction and multi-shot auctions is that if the participants do not use certificates in the current period, this auction will influence the auction in the next round. Here, we assume the participants only need to bid its value at the first period. Thus if it does not know the future demand, our auction is still truthful automatically. The system operator needs to determine the total certificate quantity they want to put into the auction. If the system operator releases too many certificates, due to the accumulation effect and the declining marginal values, the participants with certificates might avoid purchasing more. In contrast with the one-shot auction, we also take the cost of releasing the certificate into consideration as we could not release unlimited certificates. Such a releasing cost represents the punishment in the real world if the system operator releases excessive certificates but cannot collect them for carbon reduction.

This problem is challenging since we are unaware of the exact demand in the next period. Therefore, we design four heuristic strategies, emphasizing the significance of determining the release amount.

The strategies are fixed release (FR), random release (RR), proportional to last mean demand (PPMD), and reinforcement learning (RL). We simulate d with a typical two-peak distribution for each 24 hours, utilizing two Gaussian curves with the same variance and different mean. We also consider a release cost for each piece of certificate and measure the strategies' performances. The strategies are specified as follows:

- FR: Each time, we release a fixed optimal quantity of certificates based on the expected demand in one period.
- RR: Each time, we release a random quantity of certificates.
- PPMD: Each time, we calculate the optimal quantity of certificates based on the mean demand at the last 6 time slots and apply the proportion between certificate amount and mean demand in the current period.
- RL: Run a deep Q-learning framework and decide the certificate amount according to the historical demands and corresponding actions and rewards.

Concerning the detailed RL setting, we conduct a simple deep Q-learning framework with the input state of the last certificate remaining for all players and the last six periods' demand. The network is a 3-layer Multilayer Perceptron (MLP) model.

Our action for Deep Q-Network (DQN) comes from 100 discrete levels for certificate releasing, which searches in 10,100 and 1,000 levels. We also set the learning rate to be 0.001 and the batch size to be 64 in a memory capacity of 400. The Q network iterations for Q-learning is set to be 2400 steps. The action chosen proportion for exploring and exploiting is set to be 0.9. The proportion of future reward is set to be 0.6. We repeat the experiment 10 times for more convincing results.

We take 3 days with one hour resolution as an episode to train and verify the effectiveness of the four strategies. Fig. 2 summarizes the results.

Fig. 2(a) implies that RL gradually derives better performances and exceeds other strategies with the increasing training episodes. A strategic release with RL could earn about twice more than FR or RR. The average reward tends to increase after the first peak, which means RL learns a better strategy by experiencing the first peak and therefore the fluctuation for RL decreases. PPDM and RR suffer larger fluctuations due to the embedded randomness compared with FR. Fig. 2(b) shows that the release of the certificates often increases when the demand in the following period increases. It shows that RL tends to store more spare certificates when demand is slightly increasing. This observation motivates the design of PPMD, which yields better performance in Fig. 2(a) but it does not perform as well as RL. From the figure, we also find that at the first peak, the amount of the certificate release is quite different from the amounts in the following peaks, which also support the fact that RL is learning strategies at the first peak. Finally, we observe the tendency of the amount for the certificates that are still unused for a particular player. Most of the time, when the players take part in the dispatch, the certificate could be used for generation. However, there could be late nights when the demand decreases sharply, leading to many unused certificates. It again emphasizes the importance of reducing these excess useless certificates that could influence the maximal revenue of the next period's auction. Fig. 2(c) displays that our RL strategy successfully decreases the amount of excess certificates.

The results compare the effectiveness between the proposed RL strategy and other simple strategies. This framework could be a potential solution in the actual implementation of our auction in multi-shot scenes, determining an appropriate release for the quantity of the certificate. On the other hand, choosing the certificate quantity arbitrarily causes more excess certificates, which leads to revenue loss. Therefore, in the multi-shot scenario, we should plan the release in each period in detail.

6.3 Sample Complexity Verification

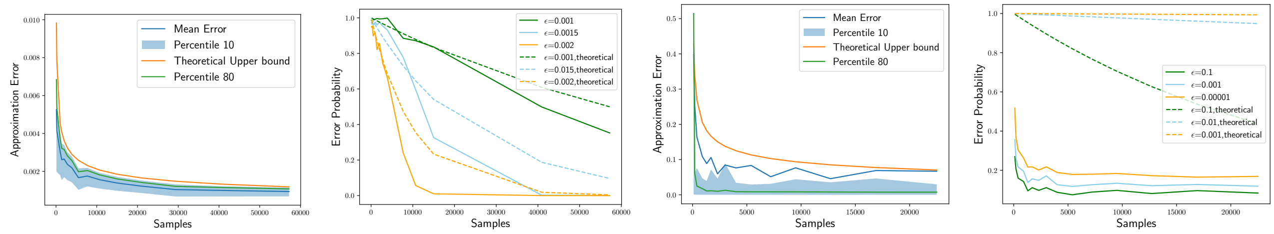
This section examines the tightness of the bound with numerical studies.

In addition to the setting mentioned above for auction and value, we assume that the value distribution's intervals are uniformly chosen. For the truncated normal distribution, the variance is symmetry and $\frac{1}{8}$ of the scale length. The probability δ in our sample complexity bound is set to be 0.1 for error ϵ's analysis. For the error probability analysis, we also set the error ϵ as different values to study the relationship between the error probability and the size of samples. For each sample size, we repeat the experiments 800 times to derive robust results.

Fig. 4 shows the numerical results. Based on our findings, the more the number of samples increases, the closer the results are to the original one for both distributions. The error decreases sharply as the sample size increases.

Based on Fig. 4(a), the theoretical curve is very close to the 90% percentile, which reflects the tightness of the proposed error bound in this case. For the normal distribution in Fig. 4(c), the theoretical bound is also higher than the 90% percentile, and empirically, it could be closer to the mean error, which shows that the theoretical results represent the mean error sometimes.

Next, we study the impact of error probability δ, which is the probability that we could not derive a good approximate auction concerning the sample number and error requirement ϵ. Obviously, the more the number of samples increases, the more the error probability decreases. In other words, the accuracy of the results increases. Moreover, the error probability increases by improving the error requirement ϵ. We take different ϵs to view the probability distribution of the errors for the approximate auction with the



(a) Mean Error for Uniform Distribution (b) Error Probability for Uniform Distribution (c) Mean Error for Normal Distribution (d) Error Probability for Normal Distribution

Figure 4: The error analysis with different size of samples

empirical distribution. Fig. 4(b) displays that our theoretical upper bound adequately describes the changes of the error probability. Regarding Fig. 4(d), the upper bound is loose, and the actual error is more minor, implying a good approximation performance even if the samples are few. We can also find out that the error probability decreases fast for the normal distribution in Fig. 4(d), which indicates that our auction that utilizes the samples can learn the samples' distribution fast and can also learn the strategies that make full use of the samples. Comparing the two distributions, we find that the bound for normal distribution could be looser than the uniform distribution. We need to afford more if we misjudge the hazard rate for normal distribution in our theoretical analysis, and our auction would also calculate a less accurate virtual demand. Overall, as shown in Fig. 4, our theoretical bound could also be loose. That is due to the large scaling in Eq. (62) and the use the uniform upper bounds and Lipschitz constants to describe the uniform characteristics of the function.

We also find that the 80% percentile is closer to the 90% percentile in Fig. 4(a) and is closer to the 10% percentile in Fig. 4(c). This implies that the error for normal distribution has higher probability to be larger than the majority of traces compared with uniform distribution. Hence, when utilizing normal distribution, we still need to collect more samples to reduce the probability of encountering the extreme samples even though the bounds perform well.

The above results show the effectiveness of our proposed theoretical sample complexity analysis and corresponding approximate auctions. The willingness of the generators is measurable without knowing any prior knowledge, and it is possible to derive a relatively good performance with the empirical distribution from samples. An increase in the scale of the samples improves results. In practice, the proposed theory shows how much error the auctioneer could have with different numbers of samples. Then the auctioneer can also decide on the number of questionnaires to ask for the values from the homogeneous participants considering error tolerance. Furthermore, more knowledge about the willingness's distribution paves the way for designing another reward mechanism for each participant's contribution.

7 CONCLUSION

We propose a framework for the green certificate auction with the call for carbon emission reduction and carbon capture technology development. Our proposed auction considers the ED process and derives truthfulness and optimality. To adequately describe the willingness of the generators to contribute to carbon neutrality, we propose to utilize sample complexity to assist our auction. We derive the upper bound of the sample number we need to derive a near-optimal outcome.

This work could be extended in many interesting directions. We plan to derive a tighter upper bound and lower bound for the sample complexity. We also intend to study the continuous complex marginal cost function in detail and other value functions that do not satisfy the assumptions in this paper. In addition, we do not specifically consider the impact of randomness in the market on our auction design. We plan to embed chance constrained optimization [29] or robust optimization [30] into our auction design to tackle this challenge.

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A PROOF FOR LEMMA 1

(**Necessary conditions**) We first prove that the Eq. (21) can be induced by the BIC and Interim IR together with an extra condition, that is $\Pi(x, v_i) \geq \Pi_i(x, x)$ for $v_i \geq x$, which implies that a bidder with a higher type, who joins the auction, could extract more for the same bidding. In the following proof, we first show $\Pi_i(v_i, v_i) \geq \Pi_i(x, v_i)$ and then $\Pi_i(v_i, v_i) \geq \Pi_i(x, x)$ for $v_i \geq x$.

We start from the following observation:

$$\begin{aligned}
&\Pi_i(v_i, v_i) - \Pi_i(v_i, x) \\
&= \mathbf{E}_{v_{-i}}\left[\Psi(q_i(v_i, v_{-i}), v_i) - \Psi(q_i(v_i, v_{-i}), x)\right] \\
&= \mathbf{E}_{v_{-i}} \int_0^{q_i(v_i, v_{-i})} \int_x^{v_i} \frac{\partial r(z, y)}{\partial y} dy dz \\
&\leq \mathbf{E}_{v_{-i}} \int_0^{q_i(v_i, v_{-i})} \int_x^{v_i} \frac{\partial r(z, x)}{\partial x} dy dz,
\end{aligned} \tag{27}$$

where the inequality follows Assumption A4. Hence for all $v_i \leq x$, we have

$$\begin{aligned}
&0 \leq \Pi_i(v_i, v_i) - \Pi_i(x, x) \\
&\quad \leq (v_i - x)\mathbf{E}_{v_{-i}} \int_0^{q_i(v_i, v_{-i})} \frac{\partial r(z, x)}{\partial x} dz
\end{aligned} \tag{28}$$

Therefore $\Pi_i(v_i, v_i)$ is continuous. In fact, $\Pi_i(v_i, x)$ is also differentiable with respect to x since $\Psi(q(x, v_{-i}), x)$ is differentiable with respect to x. Moreover, we know that $\Pi_i(x, x)$ is both continuous and non-decreasing, and hence differentiable almost everywhere. Recall the definition of BIC that

$$v_i \in argmin_x [\Pi_i(x, x) - \Pi(v_i, x)] \tag{29}$$

The first-order condition yields that:

$$\frac{d\Pi_i}{dx}(x, x) - \frac{\partial \Pi_i}{\partial x}(v_i, x) = 0 \quad \text{at} \quad x = v_i \tag{30}$$

This further indicates that

$$\begin{aligned}
&\frac{d\Pi_i}{dv_i}(v_i, v_i) = \frac{\partial \Pi_i(x, v_i)}{\partial v_i}\Big|_{x=v_i} \\
&= \mathbf{E}_{v_{-i}} \frac{\partial \Psi(q_i(v_i, v_{-i}), v_i)}{\partial v_i}.
\end{aligned} \tag{31}$$

Thus Eq. (21) directly follows the continuity of $\Pi_i(v_i, v_i)$ and Eq. (31).

(**Sufficient conditions**) Next, we prove Eq. (21) is the sufficient condition for BIC and IR.

First we observe a simple proposition that $\Pi_i(\underline{v}_i, \underline{v}_i) \geq 0$. It means that we could not force any bidder to participate and the expected surplus needs to be non-negative. Then if Π_i satisfies Eq. (21), we can further show for $y \geq x$, it holds

$$\begin{aligned}
&\Pi_i(y, y) - \Pi_i(x, x) \\
&= \mathbf{E}_{v_{-i}} \int_x^y \frac{\partial \Psi(q_i(z, v_{-i}), z)}{\partial z} dz \\
&\geq \mathbf{E}_{v_{-i}} \int_x^y \frac{\partial \Psi(q_i(x, v_{-i}), z)}{\partial z} dz,
\end{aligned} \tag{32}$$

where $q_i(z, v_{-i})$ is set as non-decreasing in z.

Due to the characteristics of r, we know that it is strictly increasing in v. Thus, $\Pi_i(v_i, v_i) \geq 0$, which is exactly Interim IR.

Hence, standard mathematical manipulation yields that for $y \geq x$

$$\begin{aligned}
&\Pi_i(x, y) - \Pi_i(x, x) \\
&= \mathbf{E}_{v_{-i}} \int_x^y \frac{\partial \Psi(q_i(x, v_{-i}), z)}{\partial z} dz.
\end{aligned} \tag{33}$$

The same trick further implies that for $y \geq x$, $\Pi_i(x, y) \geq \Pi_i(x, x)$. Together, they yield

$$\Pi_i(y, y) \geq \Pi_i(x, y) \quad y \geq x. \tag{34}$$

The almost identical argument could be made for the case when $y \leq x$, which derives the BIC conditions. ■

B PROOF FOR THEOREM 2

Lemma 1 indicates an equivalent condition for Interim IR and BIC. That is, if our designed auction satisfies Eq. (21) and the final allocation $q_i(v_i, v_{-i})$ increases with v_i, our auction could satisfy both Interim IR and BIC. Then, we only need to maximize the revenue of the auction through the allocation and payment design.

Therefore we embed the condition into Eq. (20) and derive the expression of the revenue for the system operator. More specifically, we could find that the expected payment for the bidder i is as follows:

$$\begin{aligned}\hat{p}_i(v_i) =&\mathbf{E}_{v_{-i}}[\Psi(q(v_i, v_{-i}))\\&-\int_{\underline{v}_i}^{v_i} \frac{\partial\Psi(q_i(z, v_{-i}), z)}{\partial z}dz\\&+(\lambda(v_i, v_{-i})-\alpha_i)^+ q_i(x, v_{-i})\\&+\phi(\lambda(v_i, v_{-i}), \lambda_0, \alpha_i)]-\Pi_i(\underline{v_i}, \underline{v_i})\end{aligned} \tag{35}$$

We could also derive the expected revenue for the bidder i that

$$\begin{aligned}\tilde{p}_i =&\mathbf{E}_{v_i, v_{-i}}[\Psi(q(v_i, v_{-i}))\\&-\int_{\underline{v}_i}^{v_i} \frac{\partial\Psi(q_i(z, v_{-i}), z)}{\partial z}dz\\&+(\lambda(v_i, v_{-i})-\alpha_i)^+ q_i(x, v_{-i})\\&+\phi(\lambda(v_i, v_{-i}), \lambda_0, \alpha_i)]-\Pi_i(\underline{v_i}, \underline{v_i})\\=&\mathbf{E}_{v_i, v_{-i}}[\Psi(q(v_i, v_{-i}))\\&-\frac{\partial\Psi(q_i(z, v_{-i}), z)}{\partial z}\frac{1}{\rho_i(v_i)}\\&+(\lambda(v_i, v_{-i})-\alpha_i)^+ q_i(x, v_{-i})\\&+\phi(\lambda(v_i, v_{-i}), \lambda_0, \alpha_i)]-\Pi_i(\underline{v_i}, \underline{v_i}),\end{aligned} \tag{36}$$

where $\rho_i(v_i)$ is the hazard rate.

We further design an indicator function $\omega_i(v_i, v_{-i})$ as follows:

$$\begin{aligned}&\omega_i(q_i(v_i, v_{-i}), q_{-i}(v_i, v_{-i}))\\=&\begin{cases}1 & \lambda(q_i(v_i, v_{-i}), q_{-i}(v_i, v_{-i})) \le \alpha_i\\0 & \lambda(q_i(v_i, v_{-i}), q_{-i}(v_i, v_{-i})) > \alpha_i,\end{cases}\end{aligned} \tag{37}$$

For notational simplicity, in the subsequent proof, we simplify $\omega_i(q_i(v_i, v_{-i}), q_{-i}(v_i, v_{-i}))$ to be $\omega(v_i, v_{-i})$.

This allows us to characterize the final revenue that the system operator receives denoted by R:

$$\begin{aligned}R =&\mathbf{E}_{v_i, v_{-i}}\sum_{i=1}^N [p_i-\lambda(v_i, v_{-i})\omega(v_i, v_{-i})q_i(v_i, v_{-i})\\&-\phi(\lambda(v_i, v_{-i}), \lambda_0, \alpha_i)]-\sum_{i=1}^N \Pi_i(\underline{v_i}, \underline{v_i})\\=&\mathbf{E}_{v_i, v_{-i}}\sum_{i=1}^N [\Psi(q(v_i, v_{-i}), v_i)\\&-\frac{\partial\Psi(q_i(z, v_{-i}), v_i)}{\partial v_i}\frac{1}{\rho(v_i)}\\&-\alpha_i\omega(v_i, v_{-i})q_i(x, v_{-i})]-\sum_{i=1}^N \Pi_i(\underline{v_i}, \underline{v_i})\\=&\mathbf{E}_{v_i, v_{-i}}\sum_{i=1}^N I^\lambda(q_i, v_i, \alpha_i)-\sum_{i=1}^N \Pi_i(\underline{v_i}, \underline{v_i})\end{aligned} \tag{38}$$

To maximize the revenue, we design the payment $\tilde{p}_i$ such that $\Pi_i(\underline{v_i}, \underline{v_i})$ equals 0 since we have made the proposition that $\Pi_i(\underline{v_i}, \underline{v_i}) \ge 0$.

Furthermore, we need to maximize $\sum_{i=1}^N I^\lambda(q_i, v_i, \alpha_i)$ for all v_i under the constraint $\sum_{i=1}^N q_i \le Q$, which is in optimization problem (P2). Note that we only need to find the best λ that exactly satisfies the demand d which is given by Algorithm 1. We also have shown the existence of the λ.

Finally, we demonstrate that our optimization could be well solved with the optimality conditions Eqs. (15) and $q_i(v_i, v_{-i})$ does not decrease with v_i, which is summerized in Proposition 1.

Firstly, we consider the cases that the price in ED is constant when v_i increases. Assumption A3 indicates that function I^λ is quasi-concave, indicating that for all α and λ:

$$\frac{\partial I^\lambda}{\partial q} > 0 \rightarrow \frac{1}{\rho} < \frac{r}{\frac{\partial r}{\partial q}} \tag{39}$$

Thus

$$\begin{aligned}\frac{\partial^2 I^\lambda}{\partial q^2} &= \frac{\partial r}{\partial q}-\frac{\frac{\partial^2 r}{\partial q\partial v}}{\rho} \le \frac{\partial r}{\partial q}-\frac{r\frac{\partial^2 r}{\partial q\partial v}}{\frac{\partial r}{\partial q}}\\&=\frac{r^2}{q\frac{\partial r}{\partial v}}\frac{\partial}{\partial v}\left(-\frac{q}{r}\frac{\partial r}{\partial q}\right) < 0\end{aligned} \tag{40}$$

Therefore, $I^\lambda(q, v)$ is in fact strictly quasi-concave for all α and λ.

Furthermore, we know that

$$\begin{aligned}\frac{\partial^2 I^\lambda}{\partial q\partial v} &= \frac{\partial r}{\partial v}[1-\frac{1}{\frac{\partial r}{\partial v}}\frac{\partial}{\partial v}\left(\frac{1}{\rho}\frac{\partial r}{\partial v}\right)]\\&=\frac{\partial r}{\partial v}\left(1+\frac{1}{\rho^2}\frac{d\rho}{dv}\right)-\frac{1}{\rho}\frac{\partial^2 r}{\partial v^2}\\&=\frac{\partial r}{\partial v}\frac{dJ}{dv}-\frac{1}{\rho}\frac{\partial^2 r}{\partial v^2}\\&> 0.\end{aligned} \tag{41}$$

The last inequality follows Assumptions A1 and A4. With our optimization problem (P2), we could verify the optimal solution through K.K.T. conditions. Due to strict quasi-concavity, we know Eqs. (15) are also sufficient. The remaining problem is to prove that $q^*(v_i, v_{-i})$ is non-decreasing in v_i. If $q_i^*(v_i, v_{-i}) = 0$, it trivially holds that $\frac{\partial q_i^*}{\partial v}(v_i, v_{-i}) \ge 0$ since $q_i^*(v_i, v_{-i}) \ge 0$. If $q_i^*(v_i, v_{-i}) > 0$, we could derive from Eqs. (15) that

$$\frac{\partial I^\lambda}{\partial q}(q_i^*(v_i, v_{-i}), v_i) = \mu(v_i, v_{-i}). \tag{42}$$

Differentiating with respect to v_i yields

$$\frac{\partial^2 I^\lambda}{\partial q^2}\frac{\partial q_i^*}{\partial v_i}+\frac{\partial^2 I^\lambda}{\partial q\partial v} = \frac{\partial \mu}{\partial v_i}. \tag{43}$$

Then, there are two possible conditions: if $\frac{\partial\mu}{\partial v_i}$ is non-positive, then $\frac{\partial^2 I^\lambda}{\partial q\partial v}$ is positive in Eq. (41) and $\frac{\partial^2 I^\lambda}{\partial q^2}$ is negative in Eq. (40), yielding that $\frac{\partial q_i^*}{\partial v_i}$ is positive. Otherwise, Eqs. (15) indicate that $\mu > 0$ since μ is Lagrangian multiplier. Thus for $j \ne i$, we could show if

$$\frac{\partial I^\lambda(q_j^*(v_i, v_{-i}), v_j)}{\partial q} < \mu, \tag{44}$$

then $\frac{\partial q_j^*}{\partial v_i} = 0$ since $q_j^*(v_i, v_{-i}) = 0$.

If

$$\frac{\partial I^\lambda(q_j^*(v_i, v_{-i}), v_j)}{\partial q} = \mu, \tag{45}$$

then

$$\frac{\partial^2 I^\lambda(q_j^*(v_i, v_{-i}), v_j)}{\partial q^2}\left(\frac{\partial q_j^*}{\partial v_i}\right) = \frac{\partial\mu}{\partial v_i}, \tag{46}$$

and thus $\frac{\partial q_j^*}{\partial v_i} < 0$ due to Eq. (40). In general, $\frac{\partial q_j^*}{\partial v_i}$ is non-positive. Together with our Eqs. (15) and $\mu > 0$, we conclude that

$$\sum_{j=1}^{N} \frac{\partial q_j^*}{\partial v_i} = 0, \tag{47}$$

i.e., $\frac{\partial q_i^*}{\partial v_i}$ is non-negative in this case, yielding Interim IR and BIC. The specific payment p_i is in our Eq. (36) for this scene.

Now consider the scene that the λ changes with v_i. We set the ED price as λ and the price after v_i changes to $v_i^{'}$ as $\lambda^{'}$.

If μ does not increase, it is clear that q_j^* does not decrease through Eqs. (15) and Eq. (40) for generator j whose $\alpha_j < \lambda^{'}$. Furthermore, we know that λ is non-increasing. Then if $\alpha_i < \lambda^{'}$, q_i does not decrease. For $\lambda^{'} \le \alpha_i < \lambda$, $\frac{\partial I^\lambda(q_i,v_i,\alpha_i)}{\partial q_i}$ may also increase and quantity of q_i can be non-decreasing according to Eq. (42). In addition, $\frac{\partial I^\lambda(q_i,v_i,\alpha_i)}{\partial q_i}$ of generator i does not increase if $\alpha_i \ge \lambda$. It is obvious that the quantity of q_i does not decrease.

Then if v_i and μ increase, there are four cases.

1) Assume $\alpha_i < \lambda$.

Then λ decreases as v_i increases. If λ increases, since μ decreases, the generator j with $\alpha_j > \lambda^{'}$ should be allocated less since its I^λ is constant. However, the total Q still has been allocated which means that $\sum_{\alpha_j<\lambda'} q_j$ increases and the satisfied demand must increase, which causes the contradiction.

(1.a) Suppose $\alpha_i < \lambda^{'}$. Then q_i increases. That is because q_j for $\alpha_j < \alpha_i$ will not increase as μ increases but the demand still needs to be satisfied, and $d - \sum_{\alpha_j<\lambda'} B_j$ increases then q_i^* needs to increase.

(1.b) It is evident that, $\alpha_i \ge \lambda^{'}$ does not hold. The reason is as follows: If it holds, $v_i^{'}$ may decrease to original v_i, μ will decrease, and the allocation for those generators whose $\alpha_j < \lambda^{'}$ will increase since $\frac{\partial I^\lambda(q_j,v_j,\alpha_j)}{\partial q_j}$ does not change. Then the ED price will further decrease. Hence, the demand could be also satisfied when $\lambda < \lambda^{'} \le \alpha_i$, which contradicts to our assumption.

2) Assume $\alpha_i \ge \lambda$.

The ED price increases, i.e., $\lambda^{'} > \lambda$. This is because if the price decreases, for generator j with $\alpha_j < \lambda^{'}$, its allocation will decrease and the total demand will not be satisfied, which contradicts the demand constraints in Eqs. (5).

(2.a) Suppose $\lambda^{'} \le \alpha_i$. Then function $\frac{\partial I^\lambda(q_i,v_i,\alpha_i)}{\partial q_i}$ increases in v_i and for every other generator j, its $\frac{\partial I^\lambda(q_j,v_j,\alpha_j)}{\partial q_j}$ will not increase. Then according to Eqs. (15) and (40), the allocation for i, q_i^*, will increase when μ increases for the other generators.

(2.b) Then we also need to show that the other case $\lambda^{'} > \alpha_i$ could not hold. If it holds, we could pay attention to the generator with $\alpha_j \ge \lambda'$. Its $\frac{\partial I^\lambda(q_j,v_j,\alpha_j)}{\partial q_j}$ could not change. If $v_i^{'}$ decreases to v_i, μ will decrease. Then the quantity of certificates for generator j increases and the left-over certificates decrease. We know our demand can be satisfied at $\lambda^{'}$, if the left-over certificates decrease, λ will increase and we could derive $\alpha_i < \lambda^{'} \le \lambda$, which contradicts to our assumption. Therefore, we conclude that q_i^* is monotone even if λ changes. ∎

C PROOF FOR THEOREM 3

We order the generator with respect to their marginal costs as we do in the proof for Theorem 1.

Without loss of generality, we first assume when $\lambda = \alpha_K$, we could not satisfy the demand. Then we try to set λ to be α_{K+1} and we denote original allocation for generator i as q_i and the new allocation as q_i^*. For generator K, its $I^\lambda(q_K, v_K, \alpha_K)$ will decrease by $\alpha_K q_K$. Note that when λ increases, μ associated with K.K.T conditions will not increase according to Eq. (40). Then we could derive that our allocation will be lower than before. It means that μ will not increase since $\mu \ge 0$. Since μ decreases, for generator j where $j < K$, its allocation will not decrease. There are two cases. First, if $q_j = G_j - B_j$, μ's decrease could make the Lagrangian multiplier of capacity constraint for j, τ_j, increase and q_j remains the same. Otherwise, q_j increases due to $\tau_j = 0$ and μ decreases.

Therefore, $\sum_{j<K} q_j \le \sum_{j<K} q_j^*$. Then we could further show that the maximum demand that could be satisfied $\sum_{i=1}^{K-1} B_j + q_j < \sum_{i=1}^{K} B_j + q_j$ while λ increases from α_K to α_{K+1}. We could find the maximum demand that could be satisfied is monotonic increasing and we know we could satisfy the demand if all the generators take part in the generation and all the certificates are allocated. Therefore, λ exists and it is unique.

Then we come back to the proof of Theorem 3. We need to discuss two cases. If λ is constant as v_i increases, then all I^λs are constant except for i. Hence we assume that v_i increases. Regarding Eqs. (40) and (41), I^λ is also quasi-concave, yielding a new optimal condition as follows:

$$\frac{\partial I^\lambda}{\partial q}(q_i^*(v_i, v_{-i}), v_i) = \mu(v_i, v_{-i}) + \tau_i, \tag{48}$$

where τ_i is the Lagrangian multiplier associated with the constraint $q_i \le G_i - B_i$. If $q_i^*(v_i, v_{-i}) \le G_i - B_i$, we could derive $\tau_i = 0$. The analysis is exactly the same as that in Theorem 2. If $q_i^*(v_i, v_{-i}) = G_i - B_i$, we assume there exists a $v_i^{'} > v_i$ that makes q_i^* decrease to $q_i^{*'}$. We also define other generation's allocation as $q_j^{*'}$ and $j \neq i$. We could show that

$$\begin{aligned} &I^\lambda(q_i^{*'}, v_i^{'}, \alpha_i) + \sum_{j\neq i} I^\lambda(q_j^{*'}, v_j, \alpha_j) \\ &> I^\lambda(q_i^{*}, v_i^{'}, \alpha_i) + \sum_{j\neq i} I^\lambda(q_j^{*}, v_j, \alpha_j) \\ &> I^\lambda(q_i^{*}, v_i^{'}, \alpha_i) - I^\lambda(q_i^{*}, v_i, \alpha_i) + I^\lambda(q_i^{*'}, v_i, \alpha_i) \\ &+ \sum_{j\neq i} I^\lambda(q_j^{*'}, v_j, \alpha_j), \end{aligned} \tag{49}$$

The first inequality holds due to the optimality of $q_i^{*'}$. and the second one holds due to the characteristic of I^λ. Mathematical manipulations yield that

$$\begin{aligned} &I^\lambda(q_i^{*}, v_i, \alpha_i) + I^\lambda(q_i^{*'}, v_i^{'}, \alpha_i) \\ &- I^\lambda(q_i^{*'}, v_i, \alpha_i) - I^\lambda(q_i^{*}, v_i^{'}, \alpha_i) > 0, \end{aligned} \tag{50}$$

Thus

$$\begin{aligned} &(I^\lambda(q_i^{*}, v_i, \alpha_i) + I^\lambda(q_i^{*'}, v_i^{'}, \alpha_i) - I^\lambda(q_i^{*'}, v_i, \alpha_i) \\ &- I^\lambda(q_i^{*}, v_i^{'}, \alpha_i))/((q_i^* - q_i^{*'})(v_i - v_i^{'})) < 0, \end{aligned} \tag{51}$$

If λ does not change, then I^λ is continuous. With the Lagrange mean value theorem, there exists (q, v) that makes $\frac{\partial^2 I^\lambda}{\partial q \partial v} < 0$. As we show in Eq. (41), we find the contradiction. Therefore, the monotonicity holds when λ does not change.

If λ changes, we can also prove the monotonicity. We need to analyze the cases for $q_i^*(v_i, v_{-i}) = G_i - B_i$ and $q_i^*(v_i, v_{-i}) < G_i - B_i$. We first discuss for $q_i^*(v_i, v_{-i}) < G_i - B_i$. In the following part, we denote the new ED price as $\lambda^{'}$, new type after increasing as $v_i^{'}$, and new quantity as $q_i^{*'}$ for generator i.

We first suppose μ is not increasing. Then we could show that $q_i^{*'}$ does not decrease. Since μ is not increasing, for generator j whose $\alpha_j < \lambda^{'}$, its $q_j^{*'}$ will be non-decreasing according to the K.K.T conditions of (P3). Then, the ED price will decrease, i.e., $\lambda^{'} < \lambda$ and for generator i, its $\frac{\partial I^{\lambda}(q_i, v_i, \alpha_i)}{\partial q_i}$ will be increasing according to Eq. (41). According to Eq. (40), $q_i^{*'}$ is non-decreasing.

Now we discuss the cases when μ increases. We also consider four cases.

1) Assume $\alpha_i < \lambda$.

Then we find that ED price will not increase. If the price increases, for generator j with $\alpha_j \geq \lambda^{'}$, its allocation will not increase. Provided the price increasing, we need more demand to satisfy when the allocation does not decrease for $\alpha_j \leq \lambda^{'}$, which contradicts with our demand constraints in Eqs. (5).

(1.a) Assume $\alpha_i < \lambda^{'}$. For generator j whose $\alpha_j < \alpha_i$, its $\frac{\partial I^{\lambda}(q_j, v_j, \alpha_j)}{\partial q_j}$ does not change but μ increases. Therefore, its $q_j^{*'}$ will not increase but the demand could also be satisfied and $d - \sum_{\alpha_j < \lambda^{'}} B_j$ increases. Then $q_i^{*'}$ will increase.

(1.b) We also point out the impossibility of $\lambda^{'} \leq \alpha_i$. If it holds, for generator j whose $\alpha_j < \lambda^{'}$, its $\frac{\partial I^{\lambda}(q_j, v_j, \alpha_j)}{\partial q_j}$ will not change but μ increases and the certificate quantity will decrease and the demand could not be satisfied.

2) Assume $\alpha_i \geq \lambda$.

We could make similar discussion as above that ED price will not decrease. Otherwise, for generator j with $\alpha_j < \lambda^{'}$, its allocation will not increase, which leads to the demand unsatisfied without the generation at level $\lambda^{'}$.

(2.a) Assume $\lambda^{'} \leq \alpha_i$. Then for the other generator j, $\frac{\partial I^{\lambda}(q_j, v_j, \alpha_j)}{\partial q_j}$ will not increase. According to Eq. (40), we could show when μ increases, its allocation will decrease and q_i^* will increase.

(2.b) We also show that $\alpha_i < \lambda^{'}$ is impossible. If it holds, we could derive for generator j whose $\alpha_j \geq \lambda^{'}$, its allocation will decrease and the total allocation for generator j whose $\alpha_j < \lambda^{'}$ will increase, which is higher than the original allocation for generator j whose $\alpha_j < \lambda$. Thus, the demand could be over satisfied, showing $\lambda^{'}$ could not be such high. Therefore, the monotonicity holds when $q_i^*(v_i, v_{-i}) < G_i - B_i$.

We discuss the last scene when $q_i^*(v_i, v_{-i}) = G_i - B_i$. We need to show if v_i increases, λ could not change in this case because we could not allocate more for generator i and the allocation now satisfies K.K.T conditions with higher τ_i. Hence, the q_i^* could not change in this case if v_i increases and the other generator's allocation will not change as well. We also show that the monotonicity holds when $q_i^*(v_i, v_{-i}) = G_i - B_i$.

As shown above, λ should be unique and monotonicity for the problem (P3) also holds, which implies that our mechanism could also perform well in the limited capacity cases. ∎

D PROOF FOR THEOREM 4

To prove the theorem, we first need to construct an auxiliary distribution to conduct discretization.

We first construct a finite support for each $[\underline{v}_i, \overline{v}_i]$ by the interval size of ζ, yielding $\{\underline{v}_i, \underline{v}_i + \zeta, ..., \overline{v}_i\}$ with the size of $(\overline{v}_i - \underline{v}_i)\zeta$.

We construct a discrete distribution by rounding the values from the distribution F_i to the closest multiple of ζ that is higher than the original value for $\alpha_i < \lambda$, and lower than original value otherwise. Then we denote the new distribution as $F_i^{'}$.

Now we introduce Lemma 2 to measure the error induced by the discretization:

Lemma 2 For distribution $F^{'}$, we have

$$OPT(F^{'}) \geq OPT(F) - o(\zeta), \tag{52}$$

where $OPT(F)$ denotes the optimal auction revenue under distribution F and the constant in ζ is $NL_1Q + NB_1L_2Q + NL_3B_2$ in which L_1, L_2, L_3 are Lipschitz constants and B_1, B_2 are the upper bounds for the virtual value function and the hazard rate function.

With Lemma 2, we transfer the original distribution to a simpler discrete distribution, allowing us to further construct the relationship between this new discrete and empirical distribution to complete the proof. Appendix E provides more details about the proof for Lemma 2.

Moreover, we need to utilize Theorem 1 in [25]. Lemma 3 introduces this theorem.

Lemma 3 (Theorem 1 in [25]): For any distribution $F^{'}$ on a finite set $\mathbf{v}$ such that $|v_i| \leq \kappa$ for all $1 \leq i \leq N$, suppose for some sufficiently large constant $C_2 > 0$, the number of samples is at least $C_2 \cdot \frac{Nk}{\epsilon^2} log \frac{1}{\delta}$, then with probability $1 - \delta$, for any $\mathbf{v} \rightarrow [0, 1]$, we have

$$|h(F) - h(E)| \leq \epsilon, \tag{53}$$

where E is the empirical distribution defined previously.

This allows us to map the auction results onto $[0, 1]$ with constant C_1, yielding that

$$OPT_{\mathscr{H}}(F) - \frac{NL_1Q + NB_1L_2Q + NL_3B_2}{C_1}\zeta \leq OPT_{\mathscr{H}}(F^{'}) \tag{54}$$

According to Lemma 3, denoting the sample number by m, we know that

$$|h(F^{'}) - h(E)| \leq \sqrt{C_2 \cdot \frac{N\kappa}{m} log \frac{1}{\delta}}, \tag{55}$$

with probability $1 - \delta$.

In addition, $\kappa = \max_i \frac{\overline{v}_i - \underline{v}_i}{\zeta} = \frac{C_3}{\zeta}$. Thus, the standard manipulation yields that

$$\begin{aligned} &OPT_{\mathscr{H}}(F) - \frac{NL_1Q + NB_1L_2Q + NL_3B_2}{C_1}\zeta \\ &\leq OPT_{\mathscr{H}}(F^{'}) \\ &\leq OPT_{\mathscr{H}}(E) + \sqrt{C_2 \cdot \frac{NC_3}{m\zeta} log \frac{1}{\delta}} \end{aligned} \tag{56}$$

Furthermore, Cauchy–Schwarz inequality [31] indicates that

$$OPT_{\mathscr{H}}(E) \leq OPT_{\mathscr{H}}(F) - \frac{1}{3}\left(\frac{(NL_1Q+NB_1L_2Q+NL_3B_2)C_2C_3N}{4C_1m}log\frac{1}{\delta}\right)^{\frac{1}{3}},$$

where we set

$$\zeta = \left(\frac{NC_2C_3C_1^2}{4m(NL_1Q+NB_1L_2Q+NL_3B_2)^2}log\frac{1}{\delta}\right)^{\frac{1}{3}}. \quad (57)$$

We observe that if the number of samples becomes larger, we could take higher resolution to contain the approximate error.

Thus the sample complexity of our problem is $O(\frac{N^2}{\epsilon^3}log\frac{1}{\delta})$. ■

E PROOF FOR LEMMA 2

First let M be the optimal mechanism with respect to F, which is the auction designed in Algorithm 1. Then, we construct a quantile ξ_i for each v_i. q_i satisfies $v_i(\xi_i) = \inf\{v : F_i(v) \geq \xi_i\}$ for a certain distribution F_i. If we know v, let $\underline{l}_i(v_i) = \sup_{v<v_i} F_i(v)$ and $\bar{l}_i(v_i) = F_i(v_i)$. ξ_i is uniformly sampled from $[\underline{l}_i(v_i), \bar{l}_i(v_i)]$. After deriving the mapping from v to ξ, we construct a mechanism M' for the distribution F' as follows:

- Given a rounded value $\mathbf{v}'$, we map it to get its quantile ξ for each coordinate based on the distribution F'.
- Let $\mathbf{v}''$ be the value vector that corresponds to ξ with respect to the distribution F.
- Use the mechanism M with the value vector $\mathbf{v}''$ to conduct the allocation.

The allocation is monotone for $\mathbf{v}'$, and the allocation rule in Lemma 1 guarantees the existence of a payment rule that makes M' truthful.

We further couple all the randomness by sampling the quantiles ξ. Given any ξ, we know M' and M return the same allocation. For the payment, we know that $\rho_i(v_i(\xi_i))$ and $\rho_i(v_i'(\xi_i))$ are the same. Also, the rounding process leads to a difference for value by at most ζ, and we only need to know how this difference influences the final revenue.

We focus on the characteristics of function $\Psi(q,v)$ with respect to v. We derive $\frac{\partial r(q,v)}{\partial v} > 0$ and $\frac{\partial^2 r(q,v)}{\partial v^2} \leq 0$ from our assumptions. It shows that $\Psi(q,v)$ satisfies Lipschitz continuity and the parameter is denoted by L_1. Also, $\frac{\partial \Psi(q,v)}{\partial v}$ satisfies Lipschitz continuity and the parameter is denoted by L_2.

We express the optimization problem (P4) with values $\mathbf{v}'$ as follows:

$$\text{(P4)} \max_{q_i} \sum_{i=1}^{N} I^{\lambda}(q_i, v_i', \alpha_i) \quad s.t. \quad \sum_{i=1}^{N} q_i \leq Q \quad (58)$$

Lipschitz continuity yields the following inequalities:

$$\Psi(q,v) \geq \Psi(q,v') - L_1\zeta Q \quad (59)$$

$$\frac{1}{\rho(v)} \leq \frac{1}{\rho(v')} + L_3\zeta \quad (60)$$

$$\frac{\partial \Psi(q,v)}{\partial v} \leq \frac{\partial \Psi(q,v')}{\partial v} + L_2\zeta Q \quad (61)$$

Note that we know $\frac{\partial \Psi(q,v)}{\partial v}$ is positive and it is bounded by B_2. We also have $\frac{1}{\rho_i(v_i)}$ is positive and it is uniformly bounded by B_1. Hence,

$$\begin{aligned} &\sum_{i=1}^{N} I^{\lambda}(q, v', \alpha_i) \\ &\geq \sum_{i=1}^{N} I^{\lambda}(q, v, \alpha_i) - NL_1\zeta Q - NB_1L_2\zeta Q \\ &- NL_3B_2\zeta + NB_1B_2L_2L_3\zeta^2 Q \\ &\geq \sum_{i=1}^{N} I^{\lambda}(q, v, \alpha_i) - NL_1\zeta Q - NB_1L_2\zeta Q \\ &- NL_3B_2\zeta, \end{aligned} \quad (62)$$

which corresponds to the optimization problem (P2).

Remark: There are some extreme scenarios, which change λ and the optimal revenue. That is, values after rounding for the generators that take part in the generation rise while others go down. Then we could derive that the allocation for generator j whose $\alpha_j < \lambda$ could not decrease. A decrease in allocation means that allocation for other generator i with $\alpha_i \geq \lambda$ increases. Since other $\frac{\partial I^{\lambda}(q_i,v_i,\alpha_i)}{\partial q_i}$ decreases, μ decreases. For generator j, its allocation needs to be raised where contradiction occurs. By rounding, our mechanism satisfies more demand than original values, which shows the ED price should be lower. Since we have discrete, limited price decreasing choices, the number of breaking points that $\frac{\partial q_j}{\partial v_j}$ cannot be bounded are limited. The occurrence of the price change in these breaking points has a small probability (almost zero). For other points, $\frac{\partial q_j}{\partial v_j}$ is non-negative and bounded. If ζ becomes small enough, the change of allocation is small enough and further makes Eq. (62) hold. Here, we assume that ϵ is small enough, and we could divide the interval with a small enough ζ.

We denote the optimal solution for (P4) as OPT(P4), we know

$$\begin{aligned} &OPT(F) - NL_1\zeta Q - NB_1L_2\zeta Q - NL_3B_2\zeta \\ &\leq OPT(P4) \leq OPT(F'). \end{aligned} \quad (63)$$

■

F EXTENSION FOR MORE REALISTIC SETTINGS

While in the main content, we consider a simplified setting, this setting is not that restrictive as most of the time, the power grid is not congested. Hence, the electricity pool model is already rather representative in practice. Nonetheless, we would like to provide our preliminary results on generalizing our framework to the network constrained setting with certain assumption. Denote the cost function for each generator i by $c_i(g_i)$. Then, the network constrained ED problem is as follows:

$$\begin{aligned} \text{(P5)} \min \ & \sum_{i=1}^{N} c_i(g_i) \\ s.t. \ & -h_l \leq \sum_{i=1}^{N} H_{li}g_i - H_{li}d_i \leq h_l, \forall l \\ & \sum_{i=1}^{N} g_i = \sum_{i=1}^{N} d_i \\ & 0 \leq g_i \leq B_i + q_i \quad \forall i, \end{aligned} \quad (64)$$

where H_{li}^{g} is $(l,i)^{\text{th}}$ element in the shift factor matrix to characterize the transmission constraints. To guarantee that our extension also satisfies IC, IR and optimality, we need to identify the relationship between the valuation ϕ_i and the actual revenue in ED process. We first denote α_i^m the marginal cost when $g_i = B_i$. We assume α_i^m's are ordered ascendingly. Then we make the following assumption:

A10: For the ordered maximal marginal cost α_i^m, we assume the value for ordered generator satisfies $\overline{v_i} \leq \underline{v_{i+1}}$, for all $i = 1, ..., N-1$. We further assume that function ϕ satisfies that for each $\overline{v_i}$ and $\underline{v_{i+1}}$, it holds:

$$\begin{aligned} &\frac{\partial \psi(Q, \underline{v_{i+1}})}{\partial q} - \frac{1}{\rho_i(\underline{v_{i+1}})} \frac{\partial \psi(Q, \underline{v_{i+1}})}{\partial q} \\ &- \frac{\partial \psi(0, \overline{v_i})}{\partial q} - \frac{1}{\rho_i(\overline{v_i})} \frac{\partial \psi(0, \overline{v_i})}{\partial q} \geq \alpha_N^m. \end{aligned} \tag{65}$$

Remark: This assumption relates the valuation in the auction with the cost in the ED process. The first condition in A10 can hold in real time since most of high cost generators are environmental friendly and therefore they also have high willingness to pay for the green certificate. The second condition in A10 specifies that the main incentive to buy the certificate is to maximize the generator's own valuation instead of deriving more certificates for generation. With this technique assumption, we can extend our mechanism to handle (P5). We denote the optimal solution for (P5) by $c^*(q_1, ..., q_N)$ as a function of $\{q_i, i = 1, ...N\}$, the payment for ED process under the optimal solution to (P5) by $\Lambda_i^*(q_1, ..., q_N)$, and the optimal generation by $g_i^*(q_1, ..., q_N)$, for all $i = 1, ..., N$. These allow us to adjust the decision for auction allocation, which is in corresponding to (P2):

$$\begin{aligned} \text{(P6)}\ \max_{q_i}\ & \sum_{i=1}^{N} \Psi(q_i, v_i) - \frac{1}{\rho_i(v_i)} \frac{\partial \Psi(q_i, v_i)}{\partial v_i} + c^*(q_1, ..., q_N) \\ s.t.\ & \sum_{i=1}^{N} q_i \leq Q \end{aligned} \tag{66}$$

Thus, we can design the payment as follows:

$$\begin{aligned} p_i = &\Psi(q_i^*, v_i) - \frac{1}{\rho_i(v_i)} \frac{\partial \Psi(q_i^*, v_i)}{\partial v_i} + \Lambda_i^*(q_1, ..., q_N) \\ &- \Lambda_i^*(0, ..., 0) - c_i(g_i^*(q_1, ..., q_N)) + c_i(g_i^*(0, ..., 0)). \end{aligned} \tag{67}$$

Under Assumption A1-A6 and A10, we can further prove the IC, IR and optimal for this mechanism. The proof follows the same routine as the proof to Therorem 2. The only difference is that when examining the monotonicity, we need to embed Assumption A10 to derive the result.